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Quantum interference on graphs controlled by an external electric field

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Abstract. We consider the motion of a charged quantum particle on a loop with two external leads which is placed into an electrostatic field. The loop Hamiltonian is chosen in the simplest possible way; in order to join it to the free Hamiltonians describing the leads, we employ a method based on self-adjoint extensions. Under a symmetry requirement, the resulting full Hamiltonian contains four free parameters; each junction is characterised by a pair of them. The system under consideration represents a model of metallic or semiconductor structure that can be fabricated by presently available technologies. Assuming the ballistic regime for electrons in such a structure, we calculate the resistance dependence on intensity of the external field. The results suggest the possibility of constructing quantum interference transistors whose size and switching voltage would be much smaller than in current microchips.

1. Introduction

The rapid development of the techniques of fabricating small metallic or semiconductor structures has opened up an entirely new field of research which is sometimes dubbed mesoscopic physics. The term expresses the fact that, while the structures involved are designed by experiment (i.e. in a macroscopic way), they are small enough to exhibit typical quantum effects. Conductivity measurements have been performed on various structures: rings, squares and their sequences, honeycomb networks, etc (see, for instance, Bishop *et al* 1985, Chandrasekhar *et al* 1985, Pannetier *et al* 1983, Umbach *et al* 1984, 1986). Most attention has been paid to the Aharonov-Bohm effect manifested by magnetoresistence oscillations which represent a suitable object for experimental investigation, being stable with respect to variations of the pattern geometry.

There are other interesting situations, however. It is natural to ask, for example, what happens if the structure is placed into an electric field, in particular how its resistance depends on the field intensity. Such an experiment will produce, of course, a negative result for metallic structures because of screening. On the other hand, a non-trivial effect may appear on semiconductor structures. The possibility is technically attractive, because it might open the way to a new type of switching device. In order

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to decide whether such a prospect is realistic, a careful analysis is needed. In this paper, we address ourselves to this problem.

From the microscopic point of view, the system under consideration is complicated enough to make its complete description very difficult. We shall not attempt to do it. Instead, we are going to construct a model which, as we believe, reproduces the essential features of the system, and at the same time is solvable. It is based on the assumption that the 'wires', which are building elements of the structure, are infinitely thin. Actually they represent a band of atoms whose width can be made as small as 200 Å by the technologies mentioned above. Once we replace them by lines, our problem reduces to the analysis of motion of a quantum particle which is confined to an appropriate planar graph and subjected to the electric field. We limit ourselves to the simplest non-trivial case when the graph consists of a loop with two external leads. We also assume that the external field is weak enough so that the semiclassical approximation may be used.

Let us describe briefly the contents of the paper. In the next section we show how and under which assumptions the conductivity can be calculated. The key element of the model is to describe how the electron wavefunction 'splits' at the junctions. This is discussed in § 3; it allows us to choose the Hamiltonian for a charged particle whose motion is confined to a planar curve in the presence of an external field. In the next section, the transmission coefficient is calculated. Its evaluation requires the knowledge of transfer matrices for the corresponding Schrödinger equations; we calculate them in § 5 using the semiclassical approximation. In conclusion, we give some examples of conductivity against field intensity plots which show that the prospect of constructing the above-mentioned switching devices is fully realistic.

2. The physical background

As we have said, our model is intended to describe a charged quantum particle (an electron) moving on a loop with two external leads under influence of

(*a*) a voltage U applied to the leads, and

(*b*) a homogeneous electric field of intensity parallel to the graph plane and perpendicular to the leads.

The motion on the loop is assumed to be *ballistic*, i.e. both the elastic and inelastic scattering is negligible (see figure 1).

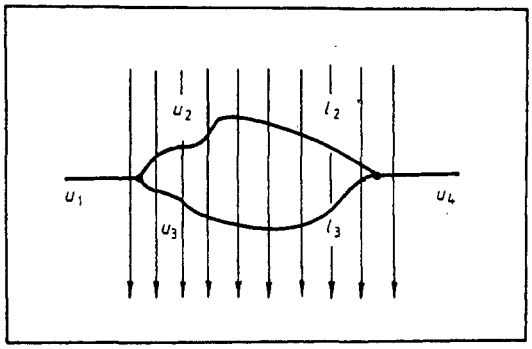

Figure 1. A loop with external leads in an electric field parallel to the graph plane: u_j and l_j are the appropriate wavefunctions and lengths of graph legs, respectively.

Such a system can be prepared, e.g., by a recently reported technology (Temkin *et al* 1987), in which one first prepares a semiconductor layer on a substrate and draws metallic lines on it lithographically. Afterwards, the graph is used as a mask when the layer is removed by ion bombarding.

Let us remark that the problem discussed here is related to that for the electrically controlled interference in sandwiched semiconductor heterostructures, which was also proposed as a basis of a switching device (or quantum interference transistor—see Datta *et al* (1986)). In this case, however, the channel width cannot be neglected, which makes the graphs more suitable; we shall return to this problem in the conclusion.

The essential assumption of our model is that the electrons move ballistically on the loop. It can be fulfilled if the electron mean free path in the semiconductor material is much longer than the loop size. This quantity depends heavily on how much the material is doped and in GaAs can be easily made a few μm long so the assumption can be justified.

The ballisticity of electron motion does not mean, of course, that it behaves as a true electron within the range of mean free path; it interacts not only with possible impurities, but with the crystalline lattice of the semiconductor material as well. By a standard solid-state physics argument, such an electron moves as a free particle with some effective mass m^*. For GaAs, e.g. we have $m^* = 0.067\, m_e$. As a consequence, the quantum conduction channels created by the transverse confinement in such a wire are widely separated. In fact, only a few of them are contained in the conductivity band (Roukes *et al* 1987, Harwitt and Harris 1987). In our model, the energy of the electron approaching the loop is assumed to coincide with the first transverse mode energy in the semiconductor wire.

Under the ballisticity assumption, one is able to solve the one-particle problem, i.e. to deduce the transmission coefficient $T(E)$ for an (effectively) free electron moving towards the loop with energy E. It also allows us to calculate the loop conductance, which is given by the formula

$$G = \frac{e^2}{\pi\hbar}\,\frac{T(E)}{1-T(E)} \tag{2.1}$$

(cf Landauer 1981); recall that $\pi\hbar/e^2 = 12\,906\ \Omega$.

3. The mathematical background

For the sake of simplicity, the electrons will be assumed to be spinless. The state Hilbert space of our problem is then of the form

$$\mathcal{H} = L^2(0,\infty)\otimes L^2(0,l_2)\otimes L^2(0,l_3)\otimes L^2(-\infty,0) \tag{3.1}$$

where the orientation of axes is chosen in such a way that allows us to describe the two junctions in a similar fashion. The central problem is how to choose the Hamiltonian H of the model. Two requirements must be fulfilled:

(a) H is self-adjoint,

(b) if the wavefunction has a support separated from the junctions, then H describes the appropriate motion on the half-line or on the loop.

Such an operator can be constructed by first taking a suitable pre-Hamiltonian H_0 which is non-self-adjoint, the branching points of the graph being removed from its domain. The admissible Hamiltonians are then obtained as self-adjoint extensions of H_0.

The same method has been applied recently in various contexts (see, e.g., Albeverio *et al* 1984, 1988, Albeverio and Hoegh-Krohn 1981, Dittrich and Exner 1985, Exner and Seba 1987, 1988a, b, c, Kuperin *et al* 1985). It may seem that by introducing the self-adjoint extension methods we burden our model with a complicated procedure which is not actually needed if one just has to write down appropriate boundary conditions connecting the wavefunctions on the legs of the graph. Even in simple cases, however, intuition may be a false guide. A fresh example (and in no sense a unique one) of such a mistake can be found in a recent paper by Bulka (1987) concerning the magnetoresistance of such a loop, where boundary conditions which look reasonable at a glance lead to a non-self-adjoint Hamiltonian, or in physical terms to probability non-conservation.

The Hamiltonians describing a free motion on a branching graph have been studied in Exner and Seba (1988c). In particular, we derived there the boundary conditions which specify the self-adjoint extensions of the pre-Hamiltonian H_0. They can be used obviously for an electron moving under the influence of an external field as long as the interaction remains bounded. This is true in our case: we shall show a little later that the starting operator H_0 may be chosen as

$$H_0 = \bigoplus_{j=1}^{4} H_{0,j} \tag{3.2a}$$

with

$$H_{0,j} = \left(-\frac{\hbar^2}{2m^*}\frac{\mathrm{d}^2}{\mathrm{d}x_j^2} + V_j(x_j) \right) \Bigg| C_0^\infty(M_j) \tag{3.2b}$$

where M_j is the appropriate part of the configuration manifold and m^* is the effective mass; furthermore, V_j is a bounded function on the loop, $j = 2, 3$, while $V_j = 0$ on the leads, $j = 1, 4$.

In general, the deficiency indices of H_0 are (6, 6) so it has a vast family of self-adjoint extensions. We adopt the following further restrictions on the Hamiltonian H:

(*c*) H is local in the sense that supp $Hu \subset$ supp u for all $u \in D(H)$,

(*d*) H is locally permutation invariant at each junction: if the support of u is sufficiently concentrated around one of the junctions, and P_{ij} is the operator permuting the jth and kth wire at the junction, then $P_{jk}u \in D(H)$ and $P_{jk}Hu = HP_{jk}u$.

This last condition means that, if the electric field is switched off, the electron whose motion is governed by H does not distinguish between the wires provided it is close enough to the junction.

Under conditions (*c*) and (*d*), the self-adjoint extensions H are characterised by two pairs of real parameters, each of them referring to one junction. Most of the extensions can be expressed by the following boundary conditions:

$$\begin{aligned} u_1(0) &= A_1u_1'(0) + B_1u_2'(0) + B_1u_3'(0) \\ u_2(0) &= B_1u_1'(0) + A_1u_2'(0) + B_1u_3'(0) \\ u_3(0) &= B_1u_1'(0) + B_1u_2'(0) + A_1u_3'(0) \end{aligned} \tag{3.3a}$$

and

$$\begin{aligned} u_2(l_2) &= A_2u_2'(l_2) + B_2u_2'(l_3) + B_2u_4'(0) \\ u_3(l_3) &= B_2u_2'(l_2) + A_2u_3'(l_3) + B_2u_4'(0) \\ u_4(0) &= B_2u_2'(l_2) + B_2u_3'(l_3) + A_2u_4'(0) \end{aligned} \tag{3.3b}$$

where $u_j(0)$, $u_j'(0)$, $u_j(l_j)$ and $u_j'(l_j)$ are understood as the limits from the appropriate side. With the chosen orientation of the axes, the two junctions are the same if

$$A_1 = -A_2 \qquad B_1 = -B_2. \tag{3.4}$$

The conditions (3.3) do not exhaust all the operators H fulfilling the requirements (a)-(d). There are two additional one-parameter classes of boundary conditions at each junction that also lead to a self-adjoint H and correspond to the cases when the conditions (3.3) become singular, namely

$$A = B - C \qquad \text{with} \qquad B \to \pm\infty \tag{3.5a}$$

for some $C \in \boldsymbol{R}$ and

$$A = \tfrac{1}{3}D - 2B \qquad \text{with} \qquad B \to \pm\infty \tag{3.5b}$$

for some $D \in \boldsymbol{R}$.

Next we must say something about the way in which the parameters specifying the junctions should be chosen. We start with the conjecture that our graph model represents a low-energy approximation to the more realistic description in which the electrons move as free particles (with the appropriate effective mass) on stripes (or tubes) instead of lines; it is suggested by the success of a similar idea for systems with point interactions (Albeverio *et al* 1988). The motion on branching stripes (or tubes) has been studied extensively in the electromagnetic and acoustic waveguide theory; the corresponding equations are the same as in the quantum mechanical case, up to the physical meaning of the spectral parameter and the wavefunctions involved.

A natural conjecture is that the parameters appearing in the boundary conditions are related to the angles specifying the junction. It is not easy, however, to find the appropriate functions by a limiting procedure with the strip width tending to zero; we are going to discuss it in a separate publication. Some conditions can be drawn, however, without a lot of mathematics. Let us compare the results obtained numerically for the so-called Y junction (Mehran 1978) with the S matrices calculated for the corresponding three-legged graph in Exner and Seba (1988c). For the Y junction, whose legs are of the same width, the zero-momentum limit is

$$\lim_{k\to 0} |S_{ij}(k)| = |\tfrac{2}{3} - \delta_{ij}|.$$

It is exactly the same as for the S matrix referring to one of the 'exceptional' boundary conditions (3.5). On the other hand, the zero-momentum limit of a Y junction with non-equivalent legs yields the total reflection, exactly as the S matrices referring to the boundary condition (3.3). Moreover, comparison of the low-energy behaviour shows, e.g., that one of the conditions (3.5) with either $C = 0$ or $D = 0$ corresponds to the totally symmetric Y junction (with 120° angle). Finally, a brief inspection of the low-energy behaviour of the S matrices corresponding to boundary conditions (3.3) shows that in practical calculations the 'exceptional' cases (3.5) may be replaced by (3.3) with A, B large enough.

The last thing we must fix are the potentials V_j appearing in (3.2b). A natural guess which can be supported by the standard quantisation procedure (Sniatycki 1980) is

$$V_j(x_j) = -e\mathscr{E}\gamma_j(x_j) \tag{3.6}$$

where $\gamma_j(x_j)$ marks the distance from a fixed equipotential line and $\mathscr{E}$ is the applied field intensity. Conventionally, we chose a zero value of the potential on the leads.

Unfortunately, the problem is more complicated. As mentioned above, the graph line substitutes thin stripes in our model and therefore the potentials should contain curvature-dependent terms (see, e.g., da Costa 1981, 1983, Jensen and Koppe 1971). Simple estimates show, however, that, for the semiconductor structures mentioned in the introduction, the ansatz (3.6) represents a good approximation. Let us remark that in other situations, curvature can produce interesting physical effects (Exner and Seba 1987).

4. The transmission coefficient

Now we are ready to solve the scattering problem for the loop. We shall use the time-independent framework; then one has to find the function $u = (u_1, u_2, u_3, u_4)$ that belongs *locally* to the domain of the extension H chosen to play the role of Hamiltonian and fulfills

$$u_1(x_1) = \exp(-ikx_1) + a\exp(ikx_1) \tag{4.1a}$$

$$u_2(x_2) = c_1 f_1(x_2) + c_2 f_2(x_2) \tag{4.1b}$$

$$u_3(x_3) = d_1 g_1(x_3) + d_2 g_2(x_3) \tag{4.1c}$$

$$u_4(x_4) = b\exp(-ikx_4) \tag{4.1d}$$

where f_1, f_2 are the solutions to the Schrödinger equation

$$-(\hbar^2/2m^*)f_k''(x_2) + V_2(x_2)f_k(x_2) = Ef_k(x_2) \tag{4.2a}$$

with $E = \hbar^2k^2/2m^*$ fulfilling the boundary conditions

$$f_1(0) = f_2'(0) = 1 \qquad f_1'(0) = f_2(0) = 0 \tag{4.3a}$$

and similarly g_1, g_2 are the solutions of

$$-(\hbar^2/2m^*)g_k''(x_3) + V_3(x_3)g_k(x_3) = Eg_k(x_3) \tag{4.2b}$$

corresponding to the boundary conditions

$$g_1(0) = g_2'(0) = 1 \qquad g_1'(0) = g_2(0) = 0. \tag{4.3b}$$

The potentials V_2, V_3 are given by (3.5) as mentioned above. We shall also need the transfer matrices $\Pi_j = \Pi_j(l_j)$, $j = 2, 3$,

$$\begin{pmatrix} u_j(l_j) \\ u_j'(i_j) \end{pmatrix} = \Pi_j \begin{pmatrix} u_j(0) \\ u_j'(0) \end{pmatrix}. \tag{4.4a}$$

Using the above-mentioned solutions to (4.2), we may express the matrices as

$$\Pi_2 = \begin{pmatrix} f_1(l_2) & f_2(l_2) \\ f_1'(l_2) & f_2'(l_2) \end{pmatrix} \qquad \Pi_3 = \begin{pmatrix} g_1(l_3) & g_2(l_3) \\ g_1'(l_3) & g_2'(l_3) \end{pmatrix}. \tag{4.4b}$$

Our aim is to find the coefficient b, assuming that the functions (4.1) fulfil the boundary conditions (3.3). To solve this problem, first we express the coefficients c_1, c_2 with the help of d_1, d_2. The boundary conditions for the first junction yield the equations

$$1 + a = -ikA_1(1-a) + c_2B_1 + d_2B_1$$

$$c_1 = -ikB_1(1-a) + c_2A_1 + d_2B_1$$

$$d_1 = -ikB_1(1-a) + c_2B_1 + d_2A_1.$$

Excluding a from here, we get a system of two linear equations for c_1, c_2 which is solved by

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} z_1(k) \\ z_2(k) \end{pmatrix} + C_1(k) \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \tag{4.5}$$

where

$$z_2(k) = (A_1 - B_1) z_1(k) \qquad z_1(k) = \frac{2ik}{1 + ik(B_1 - A_1)} \tag{4.6a}$$

and

$$C_1(k) = \frac{1}{B_1[1 + ik(B_1 - A_1)]} \times \begin{pmatrix} A_1 + ik(B_1^2 - A_1^2) & (B_1 - A_1)[(A_1 + B_1)(1 - ikA_1) + 2ikB_1^2] \\ 1 - ikA_1 & -A_1 - ik(B_1^2 - A_1^2) \end{pmatrix}. \tag{4.6b}$$

The boundary conditions at the second junction yield the equations

$$\tilde{c}_1 = \tilde{c}_2 A_2 + \tilde{d}_2 B_2 - ikbB_2$$

$$\tilde{d}_1 = \tilde{c}_2 B_2 + \tilde{d}_2 A_2 - ikbB_2$$

$$b = \tilde{c}_2 B_2 + \tilde{d}_2 B_2 - ikbA_2$$

where we have introduced

$$\begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = \Pi_2 \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} u_2(l_2) \\ u_2'(l_2) \end{pmatrix} \tag{4.7a}$$

and

$$\begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} = \Pi_3 \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} u_3(l_3) \\ u_3'(l_3) \end{pmatrix}. \tag{4.7b}$$

Excluding b from the above system:

$$b = \frac{B_2}{1 + ikA_2} (\tilde{c}_2 + \tilde{d}_2) \tag{4.8}$$

we arrive at equations for $\tilde{c}_1, \tilde{c}_2$. They are solved by

$$\begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = C_2(-k) \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} \tag{4.9}$$

where $C_2(k)$ is given by (4.6b) with A_1, B_1 replaced by A_2, B_2. Now we are in the position to find the coefficients in (4.1b, c). Notice that the matrices appearing in the above relations are non-singular; one finds easily that $\det C_j(k) = -1$, and furthermore $\det \Pi_j$ is the Wronskian of the corresponding solutions, and therefore non-zero. The relations (4.5), (4.7) and (4.9) give

$$\Pi_2^{-1} C_2(-k) \Pi_3 \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} z_1(k) \\ z_2(k) \end{pmatrix} + C_1(k) \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}.$$

From here the coefficients d_1, d_2 may be found; the other three pairs of coefficients are then obtained from (4.7) and (4.9). In particular, we have

$$\begin{pmatrix}\tilde{c}_1\\ \tilde{c}_2\end{pmatrix} = [\Pi_2^{-1} - C_1(k)\Pi_3^{-1}C_2(-k)^{-1}]^{-1}\begin{pmatrix}z_1(k)\\ z_2(k)\end{pmatrix} \tag{4.10a}$$

$$\begin{pmatrix}\tilde{d}_1\\ \tilde{d}_2\end{pmatrix} = [\Pi_2^{-1}C_2(-k) - C_1(k)\Pi_3^{-1}]^{-1}\begin{pmatrix}z_1(k)\\ z_2(k)\end{pmatrix} \tag{4.10b}$$

provided the matrices in the square brackets are non-singular (it is sufficient that one of them is non-singular). Combining the relations (4.8) and (4.10), we get the necessary transmission coefficient

$$T(E) = |b|^2 = \frac{B_2^2}{1+k^2A_2^2}|\tilde{c}_2 + \tilde{d}_2|^2 \tag{4.11}$$

where $E = \hbar^2k^2/2m^*$. In a similar way, the reflection coefficient at energy E is given by

$$R(E) = |a|^2 = \frac{|2 - B_1(c_2 + d_2)|^2}{1+k^2A_1^2}. \tag{4.12}$$

5. Semiclassical expressions for the transfer matrices

The relations (2.1) and (4.11) represent the solution to our problem. In order to calculate the conductivity, however, one must know the transfer matrices (4.4). They can be written down analytically for very few potentials, so one must look for another way. One possibility is to solve (4.2) numerically. Instead of that, we shall use here an analytically expressed but approximative solution. In fact, the WKB approximation is applicable to nearly all situations in our model with the exception of those when the energy is near the top of the potential barrier or a plateau in the 'upper' branch of the loop. We restrict our attention, however, to the simplest situation represented by the *weak-field case* when no tunnelling occurs (figure 2). This is true if

$$\mathscr{E} < E/2ae \tag{5.1}$$

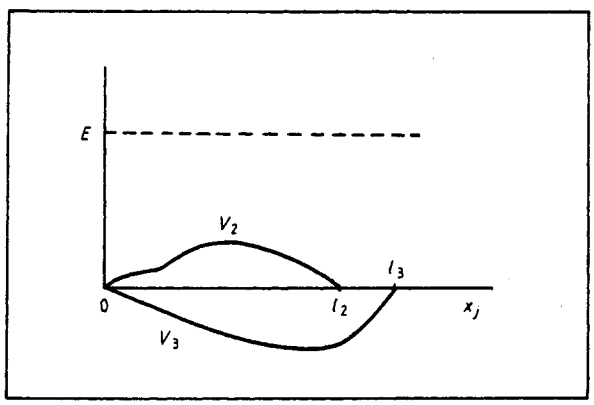

Figure 2. The potentials appearing in the Schrödinger equations (4.2).

where a is a characteristic size of the 'upper' branch, say the radius of the ring. Taking $a \approx 2 \times 10^{-5}$ cm and the typical lowest transverse-mode energy mentioned in § 2, we get $\mathscr{E} < 10^3$ V cm^{-1}; as we shall see a little later, one can obtain interference minima before the tunnelling regime takes place.

The WKB approximation may be applied if $|(\hbar/p_j(x_j))'| \ll 1$, where

$$p_j(x_j) = [2m^*(E - V_j(x_j))]^{1/2}. \tag{5.2}$$

Since $p_j(x_j) \approx (2m^*E)^{1/2}$ in the weak-field case, it yields the condition

$$\mathscr{E} \ll \frac{(2m^*E)^{3/2}}{m^*e\hbar}. \tag{5.3}$$

For E and m^* mentioned above, we therefore get the bound $\mathscr{E} \ll 10^5$ V cm^{-1} which is certainly satisfied in the weak-field case.

The general WKB solution to (4.2a) is well known; it is only necessary to select the two solutions that fulfil the boundary conditions (4.3a). We obtain

$$f_1(x_2) = \left(\frac{p_2(0)}{p_2(x_2)}\right)^{1/2} \cos\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right) - \frac{m^*\hbar V_2'(0)}{2p_2(0)^{5/2}p_2(x_2)^{1/2}} \sin\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right) \tag{5.4a}$$

$$f_1'(x_2) = m^* \frac{V_2'(x_2)p_2(0)^3 - V_2'(0)p_2(x_2)^3}{2(p_2(0)p_2(x_2))^{5/2}} \cos\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right) - \frac{(m^*\hbar)^2 V_2'(0)\,V_2'(x_2) + 4(p_2(0)p_2(x_2))^3}{4\hbar(p_2(0)p_2(x_2))^{5/2}} \sin\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right) \tag{5.4b}$$

$$f_2(x_2) = \frac{\hbar}{(p_2(0)p_2(x_2))^{1/2}} \sin\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right) \tag{5.4c}$$

$$f_2'(x_2) = -\left(\frac{p_2(x_2)}{p_2(0)}\right)^{1/2} \cos\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right) + \frac{m^*\hbar V_2'(x_2)}{2p_2(0)^{1/2}p_2(x_2)^{5/2}} \sin\left(\frac{1}{\hbar}\int_0^{x_2} p_2(y)\,dy\right). \tag{5.4d}$$

Substituting $x_2 = l_2$, and using (4.4b), we get the necessary expression of Π_2. The transfer matrix Π_3 is expressed in a similar way by means of p_3, V_3 and V_3'.

6. Conclusion

Let us now comment on the result. A typical conductance plot for a loop of the sketched shape is plotted in figure 3. In accordance with the discussion of § 3, we choose here $A_1 = B_1 = -A_2 = -B_2 = 10^4$ Å to describe the two totally symmetric junctions. We also choose $E = 0.05$ eV corresponding to a 200Å GaAs wire (Harwitt and Harris 1987) and $m^* = 0.067\ m_e$. Varying the parameters that characterise the junctions, loop shape and energy E, we obtain other curves. The qualitative character does not change, however; the conductance always exhibits large oscillations with well distinguished minima at reasonably low field intensities.

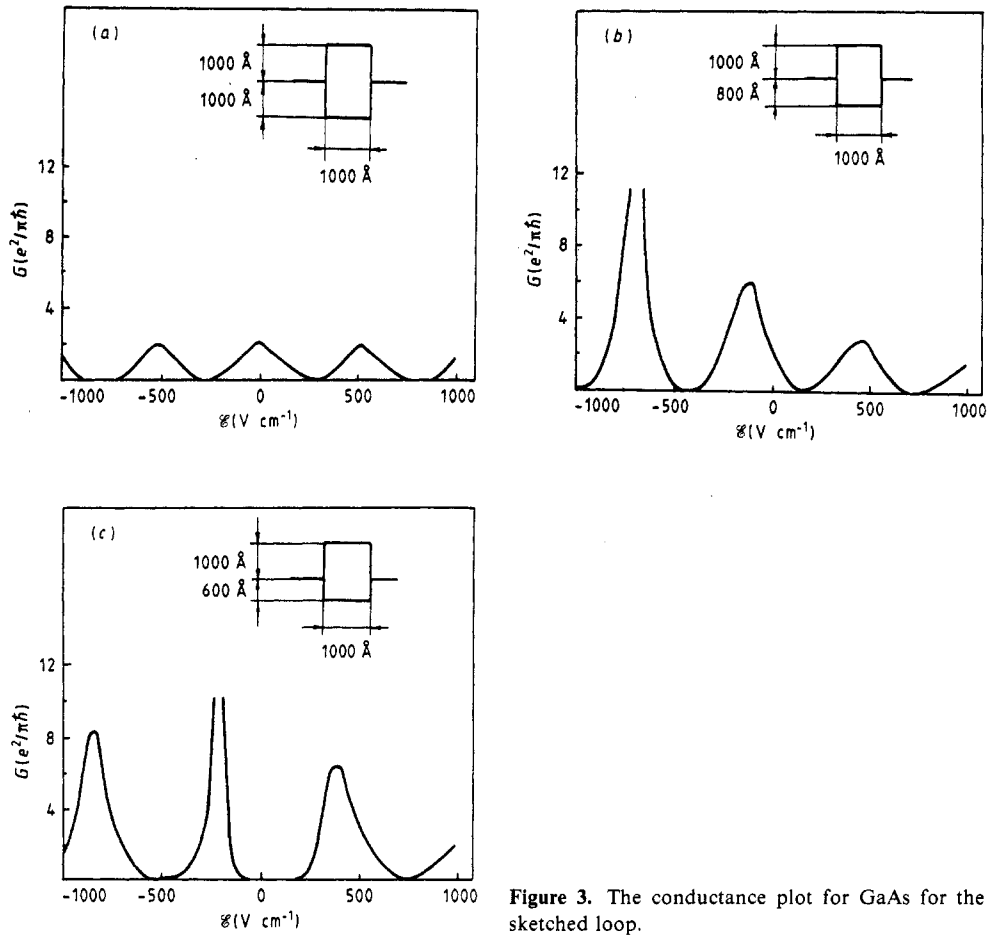

Figure 3. The conductance plot for GaAs for the sketched loop.

As we have mentioned in the introduction, our results do not apply to metallic graphs where the electrons are screened from the electric field. On the other hand, there is no screening in ultrathin semiconductor wires; this can be seen, e.g., from the experimental results obtained by Harwit and Harris (1987), where the quantum well intersubband transition in a perpendicular electric field has been investigated. The screening in the quasi-one-dimensional quantum wires has also been treated theoretically by Lai and Das Sarma (1986).

Hence we can conclude that the prospect of constructing a new type of switching device (a 'quantum interference transistor') with the controlling voltage of the order of millivolts is fully realistic. Our results show that the conductance can exhibit very large and steep modulations, in particular for some non-symmetric loop shapes, and hence provide us with a true switch-off effect. This is obviously due to the fact that the thickness of the wires is much smaller than the size of the device, which allows us to model them by infinitely thin wires. On the other hand, for systems based on heterostructures which we have mentioned in § 2, the best conductance modulations obtained theoretically in Datta *et al* (1986) have been 1:4.

The graph-type devices considered here are certainly sensitive to the loop shape, but this problem is rather a technical one and seems manageable with the high-precision

technologies available. On the other hand, an attractive possibility arises here, that one may be able to tailor the conductance plot by choosing the proper shape of loop.

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